Shannon Entropy Generalization in Tsallis Entropy Form

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Generalization of standard expression S_1 in information theory,

$S = -k\sum_{i=1}^{W} p_i \ln p_i,$ where $W \in \mathbb{N}$ is the total number of possible configurations and $\{p_i\}$ is the associated probabilities.

Tsallis postulates the entropy to

$$S_q \equiv k\frac{1-\sum_{i=1}^{W} p_i^q}{q-1} \quad q \in \mathbb{R}$$

first let's verify that

$$S_1 \equiv \lim_{q\to 1} S_q = k \lim_{q\to 1} \frac{1-\sum_{i=1}^{W} p_i \exp[(q-1)\ln p_i]}{q-1} = -k\sum_{i=1}^{W} p_i \ln p_i \ ?$$

check first and care that if we put 1 to denominator it will be $\frac{0}{0}$ or $\frac{\infty}{\infty}$ inequality.
For solving this inequality we apply L'Hopital rule $\Rightarrow$ if $\lim f(x)/g(x) = \frac{0}{0}$ or $\frac{\infty}{\infty}$ then $\lim f'(x)/g'(x)$ is equal to unsolved limit.

2

In our equation, the variable is "q". So we have to take derivative of both divider and denominator regard to "q"

$$f(q) = 1 - \sum p_i \exp[(q-1)\ln p_i]$$
$$g(q) = q - 1 \Rightarrow g'(q) = d/dq(q-1) = 1$$

numerator part's derivative is $\Rightarrow f'(q) = d/dq\{1 - \sum p_i \times \exp[(q-1)\ln p_i]\}$

since p_i and $\ln p_i$ are constants, they could be distributed into derivation.

$$f'(q) = -\sum p_i \times d/dq\{\exp[(q-1)\ln p_i]\}$$

apply chain rule that $(e^u)' = e^u \times u'$

$$u = (q-1)\ln p_i = q\ln p_i - \ln p_i$$
$$u' = \ln p_i \quad \text{(the derivation of } u \text{ regard to } q)$$

now apply

$$d/dq\{\exp[(q-1)\ln p_i]\} = \exp[(q-1)\ln p_i]\ln p_i$$

put it into $f'(q)$ equation $\Rightarrow f'(q) = -\sum p_i\{\exp[(q-1)\ln p_i] \times \ln p_i\}$

3

Now write the new limit using L'Hopital rule

$$S_1 = k \times \lim_{q \to 1} [f'(q)/g'(q)]$$

$$S_1 = k \times \lim_{q \to 1} \frac{[-\sum p_i \times \exp[(q-1)\ln p_i] \times \ln p_i]}{1}$$

take $q \to 1$ limit here $\Rightarrow S_1 = k \times \left[-\sum p_i \underbrace{\exp((1-1)\ln p_i)}_{\exp(0)=1} \times \ln p_i \right]$

$$S_1 = k \times \left[-\sum p_i \times \ln p_i \right] \quad \text{this is the generalization!}$$

it can be written as:

$$S_q = \frac{k}{q-1} \sum_{i=1}^{W} p_i (1 - p_i^{q-1})$$

References

[1] C. E. Shannon, "A Mathematical Theory of Communication," *Bell System Technical Journal*, vol. 27, no. 3, pp. 379–423, 1948.

[2] C. Tsallis, "Possible generalization of Boltzmann-Gibbs statistics," *Journal of Statistical Physics*, vol. 52, no. 1, pp. 479–487, 1988.